Entanglement-enhanced optical atomic clocks

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ABSTRACT

Recent developments in atomic physics have enabled the experimental generation of many-body entangled states to boost the performance of quantum sensors beyond the Standard Quantum Limit (SQL). This limit is imposed by the inherent projection noise of a quantum measurement. In this Perspective article, we describe the commonly used experimental methods to create many-body entangled states to operate quantum sensors beyond the SQL. In particular, we focus on the potential of applying quantum entanglement to state-of-the-art optical atomic clocks. In addition, we present recently developed time-reversal protocols that make use of complex states with high quantum Fisher information without requiring sub-SQL measurement resolution. We discuss the prospects for reaching near-Heisenberg limited quantum metrology based on such protocols.

I. INTRODUCTION

Optical-transition atomic clocks[1–5] are the most accurate sensors developed by humankind, reaching fractional stabilities below 10^{-18}. This mindboggling precision, corresponding to an uncertainty of less than one second over the age of the universe, and the associated technological improvements enable a broad range of applications in the field of precision metrology, such as the search for dark matter in the low-to intermediate-mass sector,[6–11] the investigation of nuclear structure and matter,[10,12,13] the study of any time variation of fundamental constants,[14–16] testing of the foundations of general relativity,[4,17–19] the detection of low-frequency gravitational waves,[20,21] or geodesy.[22,23] Moreover, high timekeeping precision enables improvements in navigation systems, both GPS-based and inertial.[24,25]

Currently, the main limitations to the clock precision are atomic collisions leading to atomic energy shifts,[26–31] the Dick noise[32] that is associated with the interrupted interrogation of the atomic system, and the standard quantum limit (SQL) resulting from the quantum projection noise of an ensemble of finite atom number. Schemes for removing the Dick noise have been demonstrated,[2,33–38] while collisional line shifts are usually minimized by deploying a low-density atomic gas or using engineered single-atom traps like 3D optical lattices[39,40] or arrays of optical tweezers.[41,42] Collisional shifts impose a limit on the total number of atoms that are used in optical clocks, which is typically between 10^2 and 10^4 atoms.[43] At such relatively small atom number, the SQL presents a significant constraint on the clock precision. The SQL can be overcome by engineering quantum correlations (entanglement) between the atoms.[44,45] Appropriate collective entangled states[46–54] can readily boost the performance of these advanced clocks, particularly for applications that require operation at fixed bandwidth.[6,21,55,56] It is also very appealing to apply such methods in the future to ion clocks operating with small ensembles, since entanglement methods in ion platforms are powerful and well established.[57–59]

The generation of metrologically useful entanglement on the optical clock transition of ytterbium-171 (^{171}Yb) has been recently demonstrated.[60] Yet, full clock operation beyond the SQL in a state-of-the-art optical clock represents one of the most important challenges to be met. In this article, we will give an overview and Perspective on this topic that is central to the development of future optical clocks and other sensors based on quantum interference.[45]

II. METROLOGICAL GAIN

Optical clocks measure the passing of time in terms of the frequency standard provided by an optical transition to a long-lived atomic excited state with frequency $f_a \gtrsim 100$ THz, corresponding to more than 10^{14} oscillations in a typical interrogation time of $\sim$1 s. The fractional stability of an optical clock is expressed as $\sigma \equiv \delta f / f_a$ with δf being the uncertainty of the frequency estimation.

To measure such high oscillation frequencies, one compares a local oscillator (LO) to the atoms' oscillations. The LO can be, depending on the type of operation, either an ultrastable laser[2,34,35] (complete clock implementation) or another atomic ensemble[5,61] (differential operation). The comparison is performed by measuring the accumulated phase difference between the LO and the atomic reference in a given measurement time τ (called the spectroscopy or interrogation time). In atomic clocks, this phase is mapped onto the population difference between ground and clock states. The fraction of atoms in a given state is usually measured via light absorption or fluorescence[1–5,62] or, less frequently, via cavity readout.[63] In a linear protocol, the relation between phase difference φ and frequency imbalance $\Delta f = f_a - f_{LO}$ between the LO and the atomic system is given by[64]

$$\varphi(\tau) = 2\pi \int_0^\tau w(t)\Delta f(t)\mathrm{d}t, \tag{1}$$

where $w(t)$ is the protocol sensitivity response.[64] From the accumulated phase, one can extract information about the average frequency difference. The clock stability is then limited by the precision in the estimation of the phase φ.

The goal is then to estimate the phase φ with the smallest possible uncertainty $\delta\varphi$ for a fixed set of resources, such as atom number, spectroscopy time, and dark time. The uncertainty on the phase estimation can have both classical (technical noise) and quantum contributions. In state-of-the-art optical clocks, the technical noise is of the same order or smaller than the quantum noise. For atomic sensors that employ a number N of uncorrelated atoms, the quantum noise limits the phase estimation to the SQL, $\delta\varphi_{SQL} = 1/\sqrt{N}$. The SQL is not a fundamental limit and can be overcome by proper engineering of quantum correlations (entanglement) between the atoms prior to starting the measurement protocol.[46–49]

Metrologically useful entangled states are characterized by their quantum Fisher information $\mathscr{F}_Q$,[45,65,66] which quantifies the sensitivity of the state to a change of a parameter, here a phase angle. Generally, the quantum limit to the phase estimation for a certain quantum state is given by the quantum Cramér–Rao bound,[45]

$$\delta\varphi_{CR} = \frac{1}{\sqrt{\mathscr{F}_Q}}. \tag{2}$$

Therefore, the quantum Fisher information is the figure of merit for the ultimate sensitivity achievable with a given quantum state. The field of quantum metrology investigates the class of quantum states with $\mathscr{F}_Q > N$, which is a necessary and sufficient condition for a quantum-enhanced sensor (i.e., a sensor operating beyond the SQL).[65] The fundamental limit to the phase estimation, the holy grail of quantum metrology, is the Heisenberg limit (HL) for which $\mathscr{F}_Q = N^2$.

In this article, we will discuss collective entangled states, i.e., states with $\mathscr{F}_Q > N$, and how to generate them in optical-transition clocks. We express the performances of a quantum enhanced sensors in terms of metrological gain beyond the SQL as

$$\mathscr{G} \equiv \left(\frac{\mathrm{SNR}}{\mathrm{SNR}_{SQL}}\right)^2, \tag{3}$$

where SNR and SNR_{SQL} are the signal-to-noise ratios for the actual sensor and an ideal sensor operating at the SQL, respectively. Except for relatively simple entangled states with a Gaussian envelope, known as spin squeezed states (SSSs), saturating the Cramér–Rao bound in phase estimation requires probabilistic methods beyond the standard evaluation of expectation values.[45,67,68] Hence, the metrological gain is bounded by

$$\mathscr{G} \le \frac{\mathscr{F}_Q}{N}, \tag{4}$$

where the equality applies when the Cramér-Rao bound is saturated.

III. SQUEEZING

We consider each two-level atom in the clock as a spin-$\frac{1}{2}$ system and denote by $S = N/2$, the total spin of the ensemble. Spin squeezed states (SSSs) represent a relatively simple class of collective entangled states. As shown in Fig. 1(b), in an SSS, the noise is redistributed between two orthogonal quadratures. The squeezed quadrature, with a variance (normalized to the SQL) of $\xi_-^2 = \frac{2}{S}(\Delta S_{min})^2$, is oriented along the phase axis, providing a metrological gain over the SQL by virtue of a smaller quantum noise along the phase direction. The gain $\mathscr{G}$ of a squeezed state is quantified by the Wineland parameter[46] $\xi_R^{-2} = \frac{C^2}{\xi_-^2} = \mathscr{G}$, where C is the contrast of the interferometer. As we will discuss in detail in Sec. VI, the maximum metrological gain achievable with squeezed states depends on secondary detrimental effects resulting from the quadrature with increased variance, called antisqueezing, $\xi_+^2 = \frac{2}{S}(\Delta S_{max})^2$.

Establishing any kind of quantum correlation within the atomic ensemble requires an interaction between the atoms, since they need to know about each other's state. Such an interaction can be direct, driven by state-dependent collisions or spin–spin interactions,[57,67,69,70] or effective,[50,52,53,71–73] mediated by an external field, such as a single cavity light mode interacting with all the atoms in the ensemble. Optical techniques are beneficial for metrological applications since they provide an effective atom–atom interaction that can be turned off after preparation of the spin entangled state,[52,53,71,72,74–76] thus

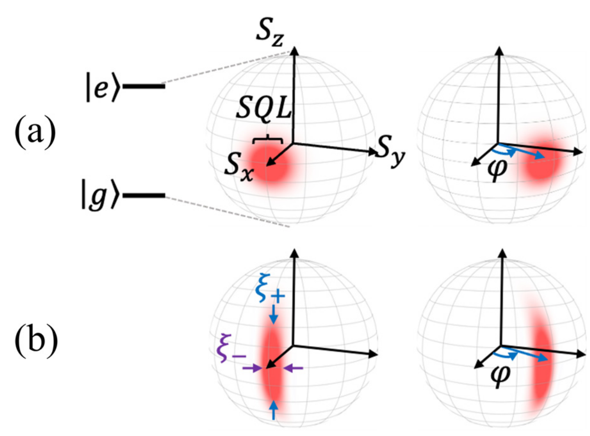

FIG. 1. Generalized Bloch Sphere Representation of an N-atom system. (a) Ramsey spectroscopy using a (not entangled) coherent spin state. The blurred circle represents the quantum projection noise that gives rise to the Standard Quantum Limit (SQL), limiting the precision of the determination of the phase φ. (b) Ramsey spectroscopy using a spin squeezed state, a special and simple collective entangled state where the quantum noise is redistributed within two orthogonal quadratures. ξ_- and ξ_+ denote the reduced (squeezed) and increased (antisqueezed) variance along two orthogonal axes, respectively. In this case, the accumulated phase φ can be estimated with precision beyond the SQL.

avoiding undesirable collision-induced energy shifts during the operation of the interferometer.

Incoherent scattering and photon loss set a bound to the maximal achievable entanglement with optical methods. Hence, the use of high-finesse cavities is preferable since they enhance the interaction relative to the scattering and have intrinsically smaller optical losses [Fig. 2(a)]. Moreover, schemes that reduce dissipation[77–81] can increase the metrological gain.

A. Measurement-based squeezing

A highly robust and conceptually straightforward technique that is able to produce record levels of spin squeezing is based on quantum nondemolition (QND) measurement,[49–54] where the uncertainty in one quadrature of the state is reduced through the measurement of photons that have interacted with the atoms [top row of Fig. 2(b)]. However, this method is necessarily imperfect, due to finite total quantum efficiency of light collection and often incomplete use of the collected information. As a result, squeezing via QND measurement in practice has operated far from unitarity, with antisqueezing significantly in excess of the minimum set by the Heisenberg uncertainty principle for a given level of squeezing.

In order to avoid incoherent spontaneous emission, so far, all cavity QED experiments aimed at the generation of collective entangled atomic states have operated far below the saturation of the atoms, i.e., in the regime of linear atomic response. In this limit, effective photon–photon interactions are weak.[81] We can then consider a coherent state of light that interacts with the atom-cavity system and remains a coherent state after that interaction. The light state $|\alpha(S_z)\rangle$ exiting the cavity, where α is the amplitude of the coherent state, depends on the S_z component of the collective atomic spin. This implies that the light field carries information about the state of the atomic ensemble. As shown by Li *et al.*,[81] this information, or light-atom entanglement, is characterized by the quantum Fisher information I of the light state

$$\tilde{I} = 4\left|\frac{d\alpha(S_z)}{dS_z}\right|^2. \tag{5}$$

In particular, the variance $(\Delta S_z)^2$ associated with estimating the parameter S_z, using knowledge of the state $|\alpha(S_z)\rangle$, is given by the relative Cramér-Rao bound $(\Delta S_z)^2 = 1/\tilde{I}$.

An external observer can gain knowledge about the S_z observable, so that the spin variance is reduced below the SQL,

$$(\Delta S_z)^2_{\text{light}} = \frac{2}{S(1+I)}, \tag{6}$$

where $I = 2\tilde{I}/S$ is the normalized Quantum Fisher information of the light field acquired by the observer ($I = 1$ resolves the SQL uncertainty). Hence, for any $I > 1$, a conditionally squeezed state of the collective atomic spin is generated.

B. Cavity feedback squeezing

As discussed above, one can generate strong conditional spin squeezing simply by quantum non-demolition measurement.[49–53] The atom-cavity interaction entangles the collective atomic spin with the light, and a high photon detection efficiency is instrumental in obtaining large amounts of spin squeezing. To eliminate the requirement of high detection efficiency, a Hamiltonian method called cavity-feedback squeezing was proposed[51] and experimentally demonstrated[72] in 2010.

The method can be understood in terms of a twofold interaction process,[71] where quantum spin fluctuations of the atomic ensemble are imprinted onto the light field that then acts back onto the atoms.[51] By placing the atoms in an optical resonator, the atom–light

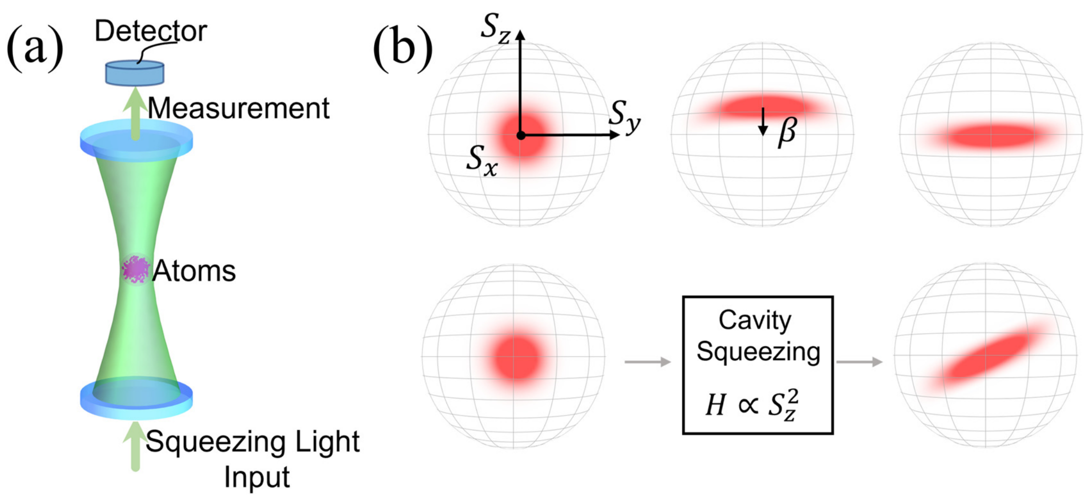

FIG. 2. Atom-cavity system for the generation of spin squeezed states. (a) An ensemble of N spin-1/2 atoms is coupled to a high-finesse optical cavity to generate the nonlinear interaction required to produce spin squeezing. (b) Top row: SSS generated by measurement-based squeezing. The squeezing is conditioned on the detection of photons that carry information about the collective spin state. After the optical measurement of the spin state via the cavity, a small pulse β can be applied to place the quantum state at the desired position on the Bloch sphere. Bottom row: deterministic cavity feedback squeezing. A CSS is subject to a one-axis twisting-like Hamiltonian ($H \propto S_z^2$) generated by the light-cavity interaction (see main text).

interaction can be significantly enhanced. In general, such strong interaction between an ensemble of N spins and the optical field of an optical resonator can be described through the Hamiltonian,[81]

$$\hat{H}_{dip} = -\hbar\Omega \hat{n}_c \hat{S}_z, \tag{7}$$

where $\hat{n}_c$ is the intracavity photon number and $\hbar\Omega$ is the light shift produced by a single photon inside the cavity. In a cavity setup, the intracavity photon number $\hat{n}_c = \hat{c}^\dagger \hat{c}$ is $\hat{S}_z$-dependent, producing to lowest order the paradigmatic one-axis twisting (OAT) Hamiltonian, initially proposed theoretically by Kitawaga and Ueda in 1993,[48]

$$\hat{H}_{OAT} = \hbar\chi \hat{S}_z^2. \tag{8}$$

This Hamiltonian produces a spin-dependent precession about the z-axis that is proportional to S_z, causing an elliptical distortion of the initial symmetric noise distribution [bottom row of Fig. 2(b)]. χ is known as the shearing parameter that quantifies how quickly the initial coherent spin state distribution is "sheared."

C. Entanglement via direct atom–atom interaction

Entanglement can also be generated by direct atom–atom interactions, e.g., collisions in a Bose–Einstein condensate,[67,82–84] the Coulomb interaction between trapped ions,[57,85] or can be turned on by promoting atoms to their Rydberg level.

Gil *et al.*[86] proposed and theoretically investigated an approach to generate an OAT Hamiltonian (8) $\hat{H} \propto \hat{S}_z^2$ directly on the optical clock transition, making use of the strong interaction between atoms in their Rydberg states. Due to the nature of the van der Waals interaction between Rydberg atoms decaying with the sixth power of the distance, the interaction Hamiltonian corresponds to an effective nearest-neighbor interaction. Recently, Schine *et al.*[87] have demonstrated pairwise entanglement of strontium atoms on the optical clock transition via Rydberg dressing, where atoms in their ground state are coupled off-resonantly to a Rydberg excited state. This work has been performed in an optical-tweezer-array clock, a new platform where atoms are individually trapped in optical tweezers and arranged in one- or two-dimensional arrays,[36,41,42] offering high tunability and control of interatomic distances.

He *et al.*[88] noted that the weak residual spin–orbit interaction in a spin-polarized Fermi gas can generate an effective OAT Hamiltonian that can be used to generate collective entangled states. In particular, they consider a 3D Fermi-degenerate optical lattice clock,[40] where spin polarized atoms are trapped in a tunable 3D optical lattice with less than one atom per lattice site and cooled down to the motional ground state (Fermi degeneracy). The spin–orbit coupling strength and, consequently, the effective non-linear collective interaction are tuned by varying the trap depth: in a very deep trap the interaction vanishes. It has been theoretically demonstrated that it is possible to generate deterministic entanglement via unitary dynamics in ensembles composed of $N \approx 10^2 - 10^4$ atoms.[88] The potential gain offered by these states can be as high as $\mathcal{G} \approx 14$ dB after an evolution time $\tau \approx 1$ s. Although this timescale may look impractically long for many clock applications, this method is particularly interesting because it makes use of usually unwanted interactions to improve the clock performance.

D. Optically generated spin squeezing in radiofrequency sensors

Measurement-based spin squeezing as well as cavity feedback squeezing have enabled the generation of entangled states with substantial amount of metrological gain in alkaline atoms.[50,52,53,72,89] Using both schemes, spin squeezing has been implemented in proof-of-principles experiments to demonstrate atomic interferometers and atomic clocks operating beyond the SQL in the radio frequency (RF) and microwave domain.[52,90,91]

Recently, cavity feedback squeezing has been demonstrated in alkaline-earth atoms,[71,76] which are of high interest for optical atomic clocks. Braverman *et al.*[76] have demonstrated a substantial amount of spin squeezing in ytterbium-171 atoms (^{171}Yb), generated via cavity feedback squeezing operating in the near-unitary regime, and offering a metrological gain of $\mathcal{G} = 6.5$ dB beyond the SQL,[76] limited by the measurement resolution of the system. The squeezing was demonstrated in the ground state of ^{171}Yb atoms, which has purely nuclear spin-1/2, representing an almost ideal two-level system to coherently manipulate the atomic state. Furthermore, using an SSS as an input state in a Ramsey protocol, a reduction in the integration time by a factor of 3.7 over the SQL was achieved.[76]

IV. MAPPING OF SQUEEZED STATES TO THE OPTICAL TRANSITION

In 2020, the first demonstration of spin squeezing on an optical clock transition was achieved.[60] The strategy used in that work was to generate an SSS in the ground state of ^{171}Yb atoms, and transfer it onto the ultra-narrow optical clock transition by applying an optical π-pulse (see Fig. 3). Then, using the SSS on the optical transition as the input state, a Ramsey protocol was demonstrated in the optical domain, where the quality factor of the transition ($Q = \Delta f/f$) was improved by five orders of magnitude compared to microwaves transitions.[43] After the Ramsey protocol was implemented in the optical domain, the squeezed state was mapped back onto the ground state and readout via the cavity.[52,60,76,92–94] Comparing the clock operations

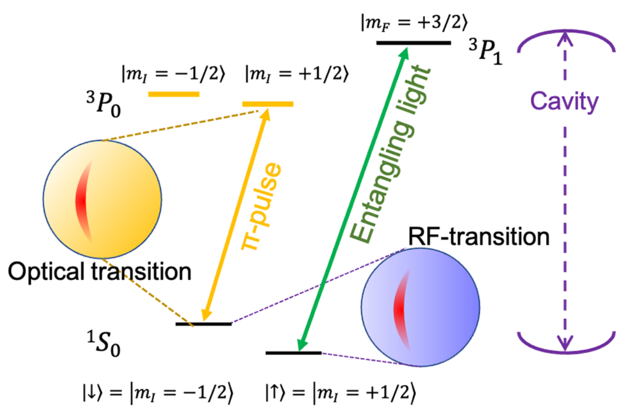

FIG. 3. Scheme for generating entanglement on the optical clock transition. The entanglement is first generated via cavity QED techniques within two atomic levels in the RF-domain, here the nuclear sublevels of the ^{171}Yb electronic ground state. Collective quantum states manipulations are easier and have very high fidelity in this domain. The entangled state is then mapped with a coherent optical π-pulse onto the optical clock transition.

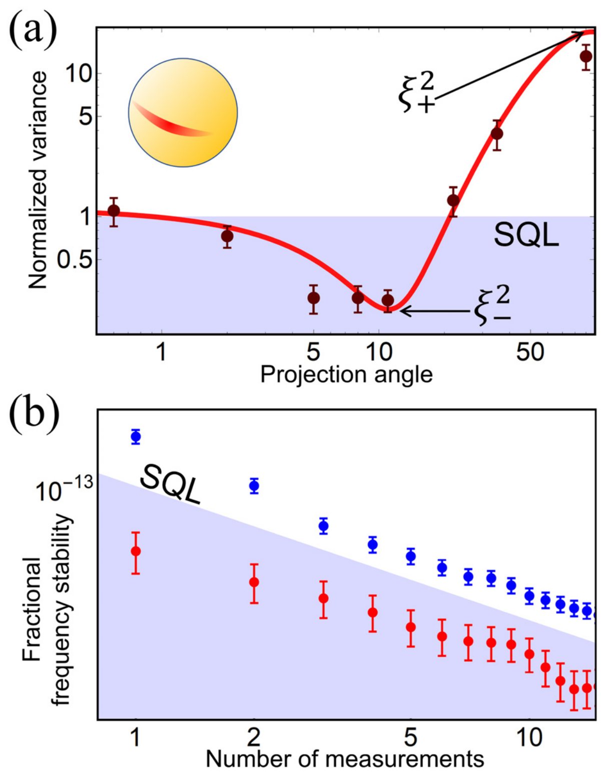

FIG. 4. Entanglement on ytterbium-171 optical clock transition from Pedrozo-Peñafiel *et al.*[60] (a) Tomography of the generated entangled state, in this case, a squeezed state, on the optical clock transition of ytterbium-171. The squeezed ξ_-^2 and the antisqueezed ξ_+^2 axes are indicated in the tomography data. (b) Allan deviation plot (fractional frequency instability) for a Ramsey sequence of the ^{171} Yb clock. Blue symbols represent the data obtained without entanglement, while red symbols correspond to a squeezed input state. The blue area is only accessible with entangled states.

using an SSS and a coherent spin state as input states, a precision of $\mathscr{G} = 4.4$ dB below the SQL was demonstrated, corresponding to a 2.8-fold reduction of the averaging time[60] (Fig. 4).

In this particular experiment, the LO stability was the limiting factor and was independently characterized and subtracted from the Allan deviation to quantify the amount of improvement with respect to the noise imposed by the atomic spins alone. In this respect, the improvement of the LO coherence is required to achieve performances at the level of state-of-the-art optical lattice clocks. In particular, optical lattice clocks use highly stable lasers that offer coherence times on the order of 1 s, corresponding to mHz linewidths.[95]

V. BEYOND SPIN SQUEEZING VIA TIME-REVERSAL PROTOCOLS

SSSs are very simple entangled states and can provide only a limited improvement over the SQL. To approach the fundamental HL where the metrological gain is $\mathscr{G} = 1/N$, it is necessary to generate more complex entangled states.[45] For example, a Greenberger–Horne–Zeilinger (GHZ) state, also called "cat-state," is an extreme example of a collective entangled state and can attain the HL. Other highly entangled states, with non-Gaussian envelopes, can carry very large quantum Fisher information $\mathscr{F}_Q$ and, hence, permit metrology close to the HL.[45,96–100] Moreover, the metrological gain of some highly entangled states follows the Heisenberg Scaling (HS),[77,96,100,101] i.e., $\mathscr{G} = b/N$ with $b \geq 1$; this class of states is a constant factor away from the HL, independent of the atom number.

Generating and utilizing highly entangled states is a difficult task due to their fragility and remains an important challenge in quantum metrology. In practice, the metrological gain is limited by the detector resolution, the curvature of the Bloch sphere, and the impossibility to access all the statistical information carried by entangled states. In particular, harnessing the resources provided by such states requires advanced statistical methods well beyond the usual evaluation of averages.

To relax those constraints, interaction-based readout methods have been proposed.[96,101–105] A particularly interesting class of protocols, which allow one to approach the HL in the realistic system, is based on Loschmidt-echo-like time-reversal of the many-body Hamiltonian.[96,101–109] Such protocols are based on the application of a many-body Hamiltonian with both positive and negative signs, corresponding to an effective evolution forward and backward in time.

A Loschmidt echo-like protocol, called Signal Amplification through Time-Reversed Interaction (SATIN),[107] is described in Fig. 5. Here, an initial coherent spin state pointing along $\hat{x}$ evolves under the action of the OAT Hamiltonian, Eq. (8), for a prolonged time to generate an over-squeezed state with large quantum Fisher information. If the process is sufficiently unitary, time reversing the Hamiltonian would produce the initial coherent spin state. However, if this entangled state is perturbed or subjected to a rotation that makes it orthogonal to the unperturbed entangled state, then the subsequent application of the negative Hamiltonian will produce a near-coherent spin state, which is now also orthogonal to the initial state. As a major advantage over direct spin squeezing, detecting the distance between these two states requires detection resolution on the order of the SQL only.

This is the feature that makes SATIN a powerful method for using highly entangled states in atom interferometry, and that enables one to reach near-Heisenberg sensitivity with Heisenberg scaling even for a many-particle system, provided the evolution of the system can be kept nearly unitary, as recently demonstrated by Colombo *et al.*[107] Specifically, highly non-Gaussian states were generated and used for atom interferometry. The SATIN protocol allowed to utilize most of the quantum Fisher information carried by these highly entangled states, reaching an improvement of 11.8 dB below the SQL (a factor of 15 in reduction of averaging time) in phase sensitivity. In addition, that protocol demonstrated HS in metrological gain $\mathscr{G}$, with a linear improvement in $\mathscr{G}$ with respect to the atom number, at a fixed distance to the HL of 12.6 dB. In the future, this protocol can be used to perform metrology in the optical domain, by mapping the entanglement onto the optical transition, as was similarly done for an SSS.[60]

Other time-reversal-like protocols have also been demonstrated in Bose–Einstein condensates through phases shifts in a three-level system for a few neutral atoms,[110] and in cold trapped ion systems consisting of up to ~150 ions, where the coupling with a motional mode plus spin rotations is used for such purpose.[85] Other approaches that mimic time-reversal-type protocols have been demonstrated, alternating spin squeezing with state rotations.[111] In particular, Hosten

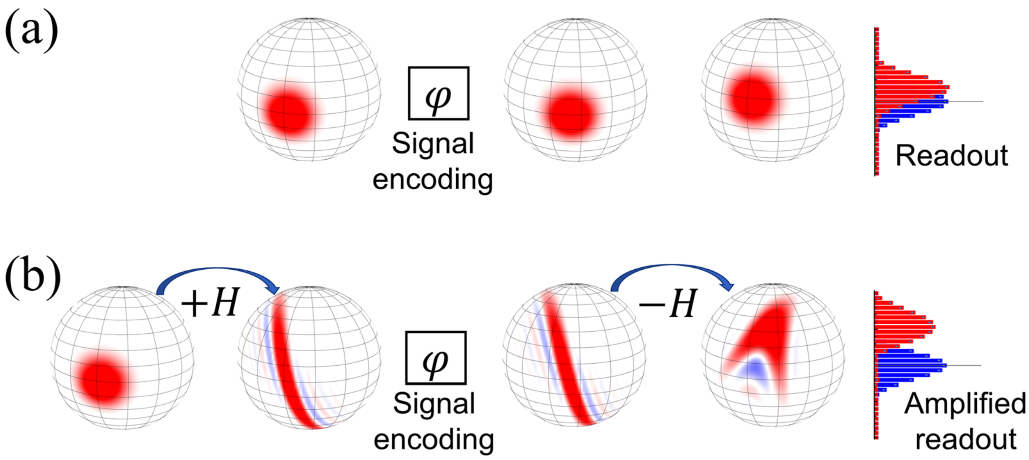

FIG. 5. Signal enhancement by SATIN protocol. The output of the measurement sequence is represented by the final state projection along the z axis. Blue histograms are the results in the absence of a signal ($\varphi = 0$). (a) Ramsey protocol with a coherent input state: the imprinted signal φ is directly reflected in the readout. (b) SATIN protocol: the signal φ is enhanced in the readout through the time-reversed application of the many-body Hamiltonian. The Fisher information contained in the small features of the highly entangled state is mapped onto a large signal after the time-reversal (–H) operation.

et al.[112] demonstrated a gain of $\mathscr{G} = 8$ dB beyond the SQL without sub-SQL measurement resolution.

The experimental demonstration of the SATIN protocol opens the door for quantum metrology with highly entangled many-body systems, paving the way to achieve nearly Heisenberg-limited operation of quantum sensors, significantly enhancing the bandwidth at fixed precision, or the precision at fixed bandwidth.

VI. USEFULNESS OF ENTANGLEMENT

In this article, we discuss mostly clocks operating with Ramsey spectroscopy.[113] In this case, the spectroscopy time is also called Ramsey time. For the Ramsey protocol, we have $w(t) = 1$, and Eq. (1) becomes $\Delta\varphi(\tau) = 2\pi\tau\langle\Delta f\rangle$, with $\langle\Delta f\rangle$ being the average frequency difference in the Ramsey time interval τ. The resulting fractional stability of a clock operated with Ramsey spectroscopy is then

$$\sigma(\tau, T, D) = \frac{1}{2\pi f_a}\sqrt{\frac{1}{\tau D T}}\frac{1}{\sqrt{N\mathscr{G}}}, \tag{9}$$

with T denoting the total measurement time and D the duty cycle of the sequence. When the clock is operated with a duty cycle D smaller than 100%, one should add the Dick noise term to Eq. (9). The latter arises from aliased noise of the LO.[32] In the absence of dark time, the Dick noise vanishes,[32,43] while it is irrelevant in applications where two or more ensembles of atoms are simultaneously probed.[2,34,35] Schulte *et al.*[114] have rigorously theoretically analyzed under which conditions entanglement can provide a precision gain in the presence of Dick noise. They found that entanglement is useful for atom numbers below a certain threshold that depends on the experimental conditions, such as the LO noise, the dark time, and the Ramsey time τ.

A. Laser as local oscillator

In full clock operation, the frequency of the laser LO is locked to the optical atomic transition, and it can be used as a time standard by means of an optical frequency comb. In a Ramsey sequence, the phase difference φ between the atoms and the LO is mapped onto the final population difference between the ground state and the clock state, i.e., $2S_z$, by means of a $\pi/2$ pulse. Since $S_z \propto \sin(\varphi)$, the protocol works correctly only if the phase accumulation does not exceed $\pm\pi/2$. Thus, as shown by Braverman *et al.*,[115] the LO noise limits the clock performance when the standard deviation of the total accumulated phase noise exceeds $\tau\Gamma_{LO} \approx 0.3$ [see Fig. 6(a)], where Γ_{LO} is the dephasing rate of the LO. At this level of noise, the probability of having a total Ramsey phase exceeding $\pm\pi/2$, and thus a wrong reading, is significant and induces an overall increase in error in the feedback.

LO noise does not induce spin–spin decoherence; however, in the case of an SSS input state, the approximation that the antisqueezing noise is orthogonal to the measurement fails. Due to the limited number of atoms utilized in optical clocks ($N \lesssim 10^4$) and the associated non-zero curvature of the generalized Bloch sphere, the anti-squeezing couples into the measurement projection, introducing extra noise and also effectively shortening the average spin vector (i.e., reducing the contrast). Larger noise and smaller contrast reduce the Wineland[46] parameter and the metrological gain.[114,116,117]

To avoid the effect of this leakage, squeezed optical clocks need to operate in a regime with a small accumulated phase φ. This implies a Ramsey time τ limited to a value smaller than that necessary to avoid phase errors exceeding $\pm\pi/2$ in an SQL clock operation. On the other hand, as can be seen from Eq. (9), the clock stability σ in a time T improves with $\sqrt{\tau}$. Therefore, to maximize clocks performance, one needs to choose a compromise between a short Ramsey time for greatest phase noise suppression and a long Ramsey time for increased clock frequency stability [see Fig. 6(a)].

Schemes that allow to profit from the full metrological gain offered by squeezed states in the presence of local oscillator noise have been proposed.[99,118] Borregaard and Sørensen[99] argue that a large accumulated phase can be strongly reduced by performing a series of QND measurements and feedback before the final strong projective

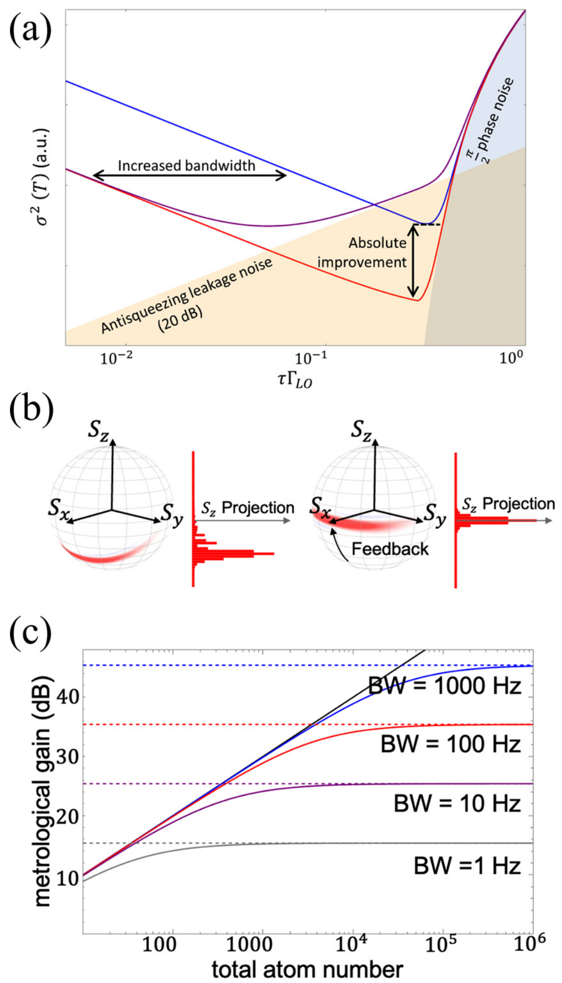

FIG. 6. Stability of optical clock in a given averaging time T as a function of Ramsey time τ. (a) Fractional stability as a function of τ in the presence of decoherence at rate Γ_{LO}. The blue, red, and purple lines indicate the operation with an unentangled coherent spin state, a unitary SSS with 11 dB of metrological gain, and a non-unitary SSS with 11 dB squeezing and 20 dB of antisqueezing, respectively. For short Ramsey time, the SSS can always outperform the coherent spin state, corresponding to an effective increase in the sensor bandwidth by $\mathscr{G}$ at fixed precision. (b) A QND measurement can be used to rotate the SSS closer to the equator, thus avoiding the leakage of the antisqueezed quadrature noise onto the measurement quadrature. (c) Maximal metrological gain achievable in a state-of-the-art system at fixed bandwidths as a function of the atom number. We considered here the decoherence rates $\Gamma_{nat} = 0.01$ s^{-1} (natural linewidth), $\Gamma_{deph} = 0.025$ s^{-1} (dephasing rate), and $\Gamma_{loss} = 0.01$ s^{-1} (atom loss rate) from the recent clock realization of Young *et al.*[42]

measurement. Compared to the case of the measurement based squeezing, here the QND measurement does not need to resolve the S_z below the SQL. It only needs to preserve the atomic coherence, i.e., contrast. The reading obtained from this weak QND measurement will be used to rotate the squeezed state closer to the equator, where the antisqueezing noise does not leak into the measurement quadrature [see Fig. 6(b)]. In a similar spirit, the large accumulated phase can be first estimated by using multiple ensembles.[118]

B. Second atomic ensemble as local oscillator

When two or more ensembles are compared, i.e., for differential operation of the sensor, the LO decoherence is a common-mode noise source and is canceled. The coherence time is then ultimately limited by the spontaneous emission rate of the atomic excited state Γ_{nat}. However, in state-of-the-art atomic clocks, other processes, such as atomic collisions and environmental inhomogeneities, can induce faster atom–atom decoherence.

Escher *et al.*[119] and Demkowicz-Dobrzański *et al.*[120] have thoroughly investigated the limitations imposed by atomic decoherence on the metrological gain. They theoretically demonstrated that, for a given uncorrelated noise, there is a maximal enhancement that can be achieved, and that such a limit depends on the Ramsey time of the sensor, i.e., on the bandwidth of the measurement.

In particular, they show that the precision in a total time T as a function of the Ramsey time τ is limited not only by the LO noise [as in Fig. 6(a)] but also by atom–atom dephasing (Γ_{deph}) and by Γ_{nat}. As illustrated in Fig. 6(a), for sufficiently short Ramsey times, the full metrological gain is recovered.

C. Quantum network of clocks

The comparison of multiple clock ensembles is the core of the idea of a quantum network of clocks, proposed by Kómár *et al.*[121] It consists of multiple, coherently interconnected clocks in which the individual devices (nodes) will benefit from the larger resources of the composed system. In particular, this approach may serve both as a way to distribute time at an international scale and a new platform for testing fundamental physics. Yet, to achieve this goal, entanglement between the different nodes is necessary to improve the performance beyond the SQL. In the best-case scenario, a Greenberger–Horne–Zeilinger (GHZ) state consisting of the product of individual GHZ states of the nodes would allow the network to operate at or near its collective HL.

A possible implementation is illustrated in Fig. 7. Two or more spatially separated ensembles can be entangled via their interaction with the same cavity mode. Each of these ensembles is effectively an individual clock and, as recently demonstrated by Zheng *et al.*,[61] they can be easily moved in and out of the cavity. In this way, the atomic ensembles can be coupled to the cavity field on demand and entangled individually as well as with each other.

Recently, Nichol *et al.*[122] have demonstrated the first quantum network of optical atomic clocks by comparing the frequencies of two entangled $^{88}Sr^{+}$ ions separated by 2 m. The entanglement between the two ions (i.e., the two clocks) was generated in a heralded fashion via a photonic link.[123]

D. Fixed-bandwidth applications

Many proposed applications of optical clocks, including gravitational-wave detection[20,21] and the search for axions,[6,7,11,124] require a large clock bandwidth and short Ramsey time, since the clock needs to track time-varying signals. Thus, for operations where the Ramsey time τ is much less than the decoherence time, $\tau\Gamma_{tot} \ll 1$, entanglement remains an important resource. In this situation, it is possible to profit from the full metrological gain offered by the entangled input state.

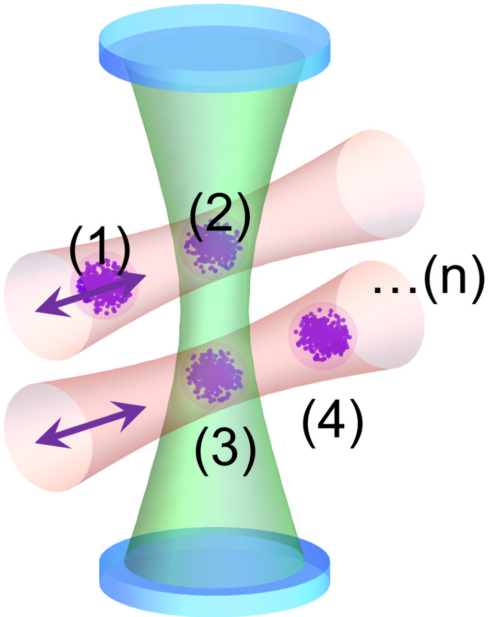

FIG. 7. Network of clocks. Several individually trapped ensembles are prepared in two optical-lattices and can be shuttled in and out of a high-finesse optical cavity. The light-mediated interaction through the cavity field can be used to engineer entanglement between the ensembles.

Figure 6(c) shows the maximum gain of an optical clock as a function of the total atom number deployed. For bandwidths set in the range 1 Hz–1 kHz, the transition from Heisenberg-limited performance to decoherence-limited performance occurs at atom numbers ranging from $\sim$50 to $\sim 5 \times 10^4$. It is worth noting that for typical atom numbers used in optical atomic clocks (10^2–10^4), a bandwidth of $\approx$10 Hz or larger implies that the maximum metrological gain is close to the Heisenberg Limit.

VII. SUMMARY AND PERSPECTIVE

In this article, we have described recent advances and offered a Perspective on strategies to improve atomic sensors via engineered quantum correlations (entanglement). The generation and application of spin squeezed states to atom interferometers and atomic clocks have enabled sub-SQL performance of these devices. While in most cases, these have been proof-of-principle experiments, some more recent implementations have shown practical applications for sensing beyond the SQL[85] in state-of-the-art sensors.

We have highlighted the generation of entanglement in optical clocks, presenting operating regimes where collective entanglement can readily bring a sensing advantage. We have also discussed a novel technique, based on effective time-reversal through switching the sign of a many-body Hamiltonian, for generating and harnessing highly entangled states (non-Gaussian states) in atomic sensors.[107] Such time-reversal protocols allow one to reach near-Heisenberg-limited quantum metrology in many-atoms sensors. This sensing paradigm strongly relaxes the most stringent limitations present in standard entangled-enhanced sensing protocols, namely, the detection of states with a large Fisher information and the curvature of the generalized Bloch sphere.

We expect that the time-reversal technique will become a major paradigm in quantum metrology in the years to come. Thanks to its applicability to non-Gaussian entangled states,[96,107] such a paradigm is ideal for the application to optical clocks and other state-of-the-art atomic sensors. Further investigations to scale up the size of entangled quantum systems as well as to reduce and tame the different decoherence mechanisms will enable near-Heisenberg limited resolution for large systems with many atoms.[119,120]

Unentangled optical clocks have achieved instabilities in relative frequency comparison of $\sigma = 6.9 \times 10^{-18}/\sqrt{T}$ using $\approx 2.4 \times 10^3$ atoms and[61] $4.4 \times 10^{-18}/\sqrt{T}$ using $\approx 2 \times 10^4$ atoms. By using entanglement and operating these state-of-the-art optical clocks in a SATIN protocol, the differential frequency instabilities could be readily pushed[107] below $10^{-18}/\sqrt{T}$. In addition to neutral-atom clocks, the stability of ion clocks could also be improved with entanglement. While usual ion clocks operate with one or two ions,[125,126] quantum computing and simulation platforms trap and entangle tens of ions.[57,127] Building upon recent advances in the trap design and control of systematic effects,[128,129] entanglement could potentially improve the stability in a 20-ion optical clock[130] by a factor of 13.

Hence, entangled optical clocks and atomic sensors have the potential to become a leading platform in the search for new physics, ranging from probing different types of physics beyond the Standard Model[55,131–134] to testing the fundamentals of gravity.[21,135,136]

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Simone Colombo: Conceptualization (equal); Data curation (equal); Formal analysis (equal); Writing – original draft (equal); Writing – review & editing (equal). **Edwin Pedrozo Penafiel:** Conceptualization (equal); Data curation (equal); Formal analysis (equal); Writing – original draft (equal); Writing – review & editing (equal). **Vladan Vuletić:** Conceptualization (equal); Data curation (equal); Formal analysis (equal); Supervision (lead); Writing – original draft (equal); Writing – review & editing (equal).

DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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