Heralded Interaction Control between Quantum Systems

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(Dated: March 3, 2020)

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SUPPLEMENTAL MATERIAL

Apparatus

The Cs atoms in our experiment are held in a far off-resonant dipole trap that is focused at the cavity waist. The trap is formed by 32 mW of 937 nm light focused through an in-vacuum lens to give an estimated transverse waist of 7 μm at the cavity mode. The corresponding approximate trap frequencies are $\omega_{\text{radial}}/(2\pi) = 6$ kHz and $\omega_{\text{axial}}/(2\pi) = 0.2$ kHz. We estimate the atoms have a radial rms radius $\sigma_{radial} = 2$ μm and an axial rms radius $\sigma_{axial} = 50$ μm. The latter is perpendicular to the cavity mode with waist $w_c = 35.5(2)$ μm.

The atom-cavity coupling g, and thus the cooperativity $\eta = 4g^2/\kappa\Gamma$, varies along the standing wave of the cavity axis and with the radial extent of the cavity mode. The position and size of the atomic cloud determine the cooperativity we realize in the experiment. The maximum cooperativity $\eta_0 = \frac{24\mathcal{F}/\pi}{k^2 w_c^2} = 8.6(1)$ is determined by the wavevector $k = 2\pi/\lambda$ where $\lambda = 852$ nm, the cavity waist $w_c = 35.5(2)$ μm, and the cavity finesse $\mathcal{F} = 77.1(5) \times 10^3$.

We model a probability distribution of the cooperativity based on the spatial distribution of the atoms. Due to pointing fluctuations of the dipole trap, along the $\hat{z}$ direction (See Fig. 2 for the coordinate system), the cooperativity varies between the maximum value of η_0 at the antinodes of the cavity standing wave and the minimum value of 0 at the nodes. For the analysis, we average over this direction to get an effective cooperativity of $\eta_0/2$. Since the dipole trap beam waist is small compared to the cavity waist, the inhomogeneous coupling effect of the atomic cloud along the $\hat{x}$ direction is negligible. The probability of a single trial to have a cooperativity $\eta(y) = \eta_0/2 \exp\big(-2y^2/w_c^2\big)$ is then determined by the spatial distribution of the cloud along the $\hat{y}$ direction $p(\eta(y)) = \exp\big(-y^2/2\sigma_{axial}^2\big)$.

Projected gain and probability of success

As we have shown in main text, we project the entangled state $|\Psi\rangle$ onto some chosen polarization of the cavity ancilla photon $|\beta\rangle_A = \cos(\theta/2)\,|\sigma^-\rangle_A + \sin(\theta/2)e^{i\varphi}\,|\sigma^+\rangle_A$. The Poincaré sphere coordinates θ and φ are decided by the angles of half-(HWP) and quarter-

(QWP) waveplates after the cavity, θ_h, θ_q,

$$\tan\frac{\theta}{2}e^{i\varphi} = \frac{-i + e^{i2\theta_q}}{ie^{i4\theta_h} - e^{i2(2\theta_h-\theta_q)}}. \tag{S1}$$

In the main text, we consider the cut where $\varphi = \pi$. This corresponds to the condition of $2\theta_h = \theta_q$.

The gain introduced to the signal coherent state after projection is $G = |\alpha'/\alpha|^2 = \langle n_S|_{n_A=1}\rangle / \langle n_S|_{n_A=0}\rangle$, where notation $\langle n_a|_{n_b=k}\rangle$ stands for the average photon counts of mode a conditioned on having k photon detected in mode b. We show here that to the lowest order, this equals to the cross correlation function $g^{(2)}_{SA}$ measured between signal and ancilla fields,

$$\begin{aligned} g^{(2)}_{SA} &= \frac{\langle n_S|_{n_A=1}\rangle \langle n_A\rangle}{\langle n_S\rangle \langle n_A\rangle} = \frac{\langle n_S|_{n_A=1}\rangle}{\langle n_S\rangle} \\ &= \frac{\langle n_S|_{n_A=1}\rangle}{p(n_A=0)\langle n_S|_{n_A=0}\rangle + p(n_A=1)\langle n_S|_{n_A=1}\rangle}. \\ &= \frac{G}{p(n_A=0) + p(n_A=1)G} \end{aligned} \tag{S2}$$

where $p(n_A = 1)$ stands for the success probability of detecting a single ancilla photon in the chosen polarization, while $p(n_A = 0)$ is the probability that no photon is detected. In the limit of weak coherent ancilla state, we have $p(n_A = 0) + p(n_A = 1) = 1$.

From Eq. S2 we get the expression for the gain G:

$$G = \frac{p(n_A=0)g^{(2)}_{SA}}{1 - p(n_A=1)g^{(2)}_{SA}}. \tag{S3}$$

In this experiment, we tune the parameters so that the success probability is usually low (Fig. 4). Thus we have $p(n_A = 0) \approx 1$ and $p(n_A = 1)g^{(2)}_{SA} \ll 1$. In this lowest order approximation, $G \approx g^{(2)}_{SA}$.

Considering the effect of background counts due to the polarization impurity of the ancilla photons, the measured gain G_{exp} is given by

$$G_{exp} = \frac{g^{(2)}_{SA} \times SNR + 1}{SNR + 1} = \frac{G \times SNR + 1}{SNR + 1}, \tag{S4}$$

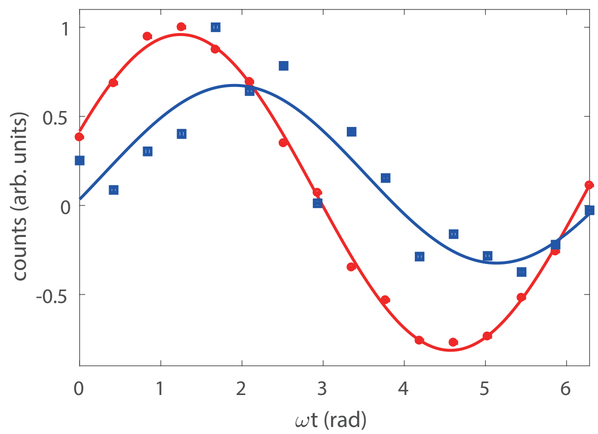

FIG. S1. **Signal phase reconstruction** The signal coherent state is mixed with a 30-MHz-detuned phase reference. By extracting the phase of the beatnote, we reconstruct the phase of the signal state. The red circles and blue squares and the corresponding fitting curves represent the sinusoidal beatnote for the signal state before and after the conditional interaction, respectively.

where SNR is the signal-to-noise ratio on the ancilla path that varies with projection basis and is given by (ignoring detector dark counts) $SNR = p(n_A = 1)/\epsilon p(n_A = 0)$. Here $\epsilon \simeq 2\%$ is the measured polarization purity limited by atomic birefregence induced by run-by-run atom number fluctuation. Due to this technical imperfection, the measure gain will be averaged towards 1 (no amplification).

To calculate the success probability of projecting onto the target signal state we measure the average photon number in the ancilla port and normalize it to the total ancilla photon number exiting both ports of the PBS after the cavity.

Signal phase reconstruction

To measure the phase of the signal field $\theta = \mathrm{Arg}(\alpha')$, we mix the signal photons with a phase reference pulse which is detuned by 30 MHz from the atomic resonance. By measuring the phase of the sinusoidal beat note between them, we reconstruct the phase of the outgoing signal coherent state (Fig. S1).

There are technical imperfections that limit the observation of the phase experimentally. Below we list those technical limitations and explain how their effect is included in the model.

- *Inhomogeneous coupling of the cavity light to the atoms.* As the atomic ensemble extends beyond the cavity mode and the atoms are distributed between nodes and anti-nodes of the cavity, different atoms couple to the cavity with different strengths. As discussed, we model this by calculating the averaged single atom cooperativity. However, when it comes to the signal phase, the varying coupling strength between the atom and the cavity also leads to a variation of the retrieved photon phase, and hence to a reduction in the contrast of the beatnote. The larger the imprinted phase is, the larger this effect will be.

- *Atom number fluctuation and atom loss during measurement.* Each storage-interaction-retrieval experiment takes 6 μs that we repeat for 30 ms before we drop, reload the MOT and repeat the experiment cycle. During this 30ms period, due to limited life time of the atoms in the trap, the optical density decreases linearly. This shifts the cavity resonance frequency, as well as changes the birefringence induced by the atoms onto the ancilla light. These imperfections appear as an effective change in the waveplate angle over 30 ms. Similarly, atom number fluctuation due to the loading noise, can be modeled as a random variation of the waveplates' angle during measurement. These effects can be accounted for by averaging fringes of different phase resulting from a small variation in the projection angle. These effects have not been included in the model shown in Fig. 3.

- *Background detection counts.* The signal-to-noise ratio on the ancilla detection path varies with the projection angle. Projection to a polarization with Poincaré sphere coordinates $\theta = \pi/2$, $\varphi = \pi$ corresponds to the lowest ancilla detection rate. When background counts are comparable to the true detection events, the phase of the projected state is mixed due to the false background events and can be estimated as:

$$\phi = \mathrm{Arg}(P_{bg} + (1 - P_{bg})e^{i\theta}), \tag{S5}$$

where $P_{bg} = \frac{\epsilon p(n_A=0)}{\epsilon p(n_A=0)+p(n_A=1)}$ is the normalized probability of having a background click.

We note that ancilla-induced loss does not change the phase but reduces the amplitude of the signal interference fringe.

Concurrence estimation

We provide an estimation on the concurrence of the two light modes. After the interaction, the joint state $|\Psi\rangle$ is given by Eq. 1 from main text. The average photon number in the ancilla pulse is measured to be $\langle n_c \rangle = 0.2$. We can then estimate the density matrix of the joint quantum system to be:

$$\rho = n_c |\Psi\rangle \langle\Psi| + (1 - n_c) |\Psi\rangle_0 \langle\Psi|_0 , \tag{S6}$$

where $|\Psi\rangle_0$ is the joint state without interaction:

$$|\Psi\rangle_0 = (|0\rangle_S + \alpha |1\rangle_S) \otimes (|\sigma^+\rangle_A + |\sigma^-\rangle_A). \tag{S7}$$

We can then extract the concurrence C of the two modes:

$$C = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}). \tag{S8}$$

where λ_i $(i = 1, 2, 3, 4)$ are the eigenvalues of matrix $\rho(\sigma_{y,S} \otimes \sigma_{y,A})\rho^T(\sigma_{y,S} \otimes \sigma_{y,A})$, sorted in reverse order [1]. Here $\sigma_{y,S}$ and $\sigma_{y,A}$ are the Pauli y matrices for the signal and ancilla mode, respectively. Substituting the numbers, we have $C = 0.11$.

In the treatment above, we have not considered any technical imperfections that limit the purity of the state. Therefore, the result of $C = 0.11$ is an upper bound of the concurrence of the entangled system.

Higher photon number states

In the main text, we have limited our discussion in the weak coherent state regime. In this section we look into the contribution of the higher photon number states.

We now keep all terms in the input signal coherent state:

$$|\phi\rangle_{S,in} = |\alpha\rangle_S = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle_S \tag{S9}$$

Entangled with the ancilla cavity photon, the (unnormalized) state is written as:

$$|\Psi\rangle = |\sigma^-\rangle_A \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle_S + |\sigma^+\rangle_A \sum_{n=0}^{\infty} t_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle_S \tag{S10}$$

where $t_n = 1/(1 + n\eta)$ is the transmission amplitude of the cavity photon corresponding to n excitations in the atom ensemble.

By conditioning on detecting the ancilla photon in basis $|\beta\rangle_A = \cos\frac{\theta}{2} |\sigma^-\rangle_A + \sin\frac{\theta}{2} e^{i\varphi} |\sigma^+\rangle_A$, we have the projected signal state:

$$|\phi\rangle_{S,out} \propto \sum_{n=0}^{\infty} C_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle_S \tag{S11}$$

where the projection coefficient C_n is written as:

$$C_n =_A \langle\beta| (|\sigma^+\rangle_A + |\sigma^-\rangle_A) = \cos\frac{\theta}{2} + t_n \sin\frac{\theta}{2} e^{i\varphi} \tag{S12}$$

Considering the high cooperativity limit where $\eta \to +\infty$, the transmission amplitude is simplified as:

$$t_n = \begin{cases} 1, & \text{if } n = 0 \\ 0, & \text{otherwise} \end{cases} \tag{S13}$$

Plugging Eq. S13 and Eq. S12 into Eq. S11, we express the projected signal state as:

$$|\phi\rangle_{S,out} \propto e^{\frac{-|\alpha|^2}{2}} \tan\frac{\theta}{2} e^{i\varphi} |0\rangle_S + |\alpha\rangle_S \tag{S14}$$

Eq. S14 tells the nature of the interaction. The projection tailors the vacuum part of the input coherent state. When $\tan(\theta/2)e^{i\varphi}$ gets close to -1, the vacuum component of the input state gets suppressed in its amplitude. In the limit of weak coherent state $|\alpha| \ll 1$, this is equivalent to amplifying the state. Similarly, in this limit, a modification to the vacuum component's phase is equivalent to a phase shift to the signal state. However, when $|\alpha|$ gets larger, the effective boost of the mean photon number in the signal state gets weaker and the output signal state deviates from a coherent state. The phase between the $|1\rangle_S$ component and the $|0\rangle_S$ component does not stand for the phase of the output signal state anymore. In the limit of large $|\alpha\rangle$, the modification from the interaction is negligible, no matter which basis $|\beta\rangle_A$ we choose to measure. These limitations are in consistent with the theoretical conclusion that phase-insensitive noiseless amplification could only happen with small input signals [2].

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