Collective State Measurement of Mesoscopic Ensembles with Single-Atom Resolution: Supplementary Information

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I. CAVITY AND DIPOLE TRAP PARAMETERS

The parameters of the near-confocal cavity at the wavelength of the probe light are summarized in Table A1. The characteristics of the atomic cloud in the optical dipole trap are summarized in Table A2. These parameters are known from previous measurements of the apparatus more fully described in [1].

cavity length	L	26.62(1)mm
Free spectral range	$\omega_{\rm FSR}/(2\pi)$	5632.0(2)MHz
Finesse	$\mathcal{F}$	$5.6(2)\times 10^3$
Linewidth	$\kappa/(2\pi)$	1.01(3) MHz
Mode waist	w	56.9(4)μm

TABLE A1: Cavity parameters. All values refer to the probe wavelength $\lambda = 780$ nm. The mode waist is calculated at the position of the atom cloud.

Trap depth	U_0/h	18(3) MHz
Mode waist	$w_{\rm t}$	59.5(5)μm
Radial frequency	$\omega_{\rm r}/(2\pi)$	1.5(1)kHz
Atom radial temperature	$k_{\rm B}T_{\rm r}$	1.0(1)MHz
Atomic cloud rms radius	$\rho_{\rm rms}$	7.0(7)μm

TABLE A2: Parameters of the optical dipole trap and the atom cloud.

II. VERIFICATION OF SINGLE-ATOM SIGNAL FROM THE VARIANCE OF A BINOMIAL DISTRIBUTION

When an average fraction f of atoms is depumped from an ensemble containing N_0 atoms, the measured variance of the depumped atom number ${\rm Var}(N_{0\rm d})$ follows the binomial distribution ${\rm Var}(N_{0\rm d}) = N_0 f(1-f)$. Next we express this binomial distribution in terms of the antinode atom number N and the depumped antinode atom number $N_{\rm d}$. Because N_0 atoms are uniformly distributed along the standing-wave of the probe light, the effective antinode atom number is $N = \langle \cos^2 kx\rangle N_0$, where $\langle\rangle$ denotes an average over the atomic position x along the cavity axis. Atom number fluctuations at different positions x are uncorrelated, so the variance of $N_{\rm d}$ is the integral of the depumped atom number variance at position x weighted by $\cos^4 kx$, giving ${\rm Var}(N_{\rm d}) = \langle \cos^4 kx\rangle {\rm Var}(N_{0\rm d})$. Hence we obtain the variance of the depumped antinode atom number

$$
{\rm Var}(N_{\rm d}) = \frac{\langle \cos^4 kz\rangle}{\langle \cos^2 kz\rangle} f(1-f)N = \frac{3}{4} f(1-f)N. \qquad (1)
$$

The cavity shift is $\delta\omega = \alpha(g^2/\Delta)N$, where α takes into account the reduction of the atom-cavity coupling due to the finite radial temperature and is given by the thermal average

$$
\alpha = \frac{\int_0^\infty {\rm d}r r e^{-2r^2/w^2} e^{-m\omega_{\rm r}^2 r^2/2k_{\rm B}T_{\rm r}}}{\int_0^\infty {\rm d}r r e^{-m\omega_{\rm r}^2 r^2/2k_{\rm B}T_{\rm r}}} = 0.94 \qquad (2)
$$

for our atomic cloud.

The variance of the change in the cavity shift caused by depumping N_d atoms is ${\rm Var}(\delta\omega_d) = \alpha^2(g^2/\Delta)^2 {\rm Var}(N_{\rm d})$. Here we neglect the fluctuation of the radial coupling because α is close to unity, and our typical measurement time is the same or larger than the period of the radial motion thus the measurement averages over different radial positions. Substituting Eq. 1 we obtain the normalized variance

$$
V = \frac{{\rm Var}(\delta\omega_d)}{\delta\omega} = \frac{3}{4}\alpha\frac{g_0^2}{\Delta} f(1-f). \qquad (3)
$$

III. DEPENDENCE OF THE VARIANCE ON AVERAGING TIME

Equation 1 in the text describes the dependence of the measured atom variance on averaging time τ. Several sources contribute to the measured variance. These include shot noise fluctuations in the probe laser power, additional technical noise due to probe laser frequency fluctuations, and the shot noise of atom loss from the state of interest due to Raman scattering events or evaporation from the trap. At a given atom-cavity detuning Δ and probe power P, the contributions to the variance due to laser noise can be combined into a single term which averages down with τ, while those related to atom loss are combined into a single term which increases with τ. For the laser powers and atom-cavity detunings used for these measurements, electronic noise in our detector makes a negligible contribution to our measurement variance. We also looked for a fixed contribution to the measurement variance (i.e., a contribution not scaling with time τ), but found this term to be negligible.

The measured atom number variance is thus described as

$$(\Delta N)^2 = c_1\tau^{-1} + c_2 N\tau, \tag{4}$$

with c_1 and c_2 constants. As c_1 depends only on laser fluctuations, this term can be determined by a measurement of the signal fluctuations when no atoms are present in the cavity (and, therefore, no contribution from atom loss is present). The fit shown in Figure 2 establishes c_1, in terms of an atom number variance at a detuning of $\Delta/(2\pi) = 250$ MHz, to be 0.17 ms. The coefficient c_2 must be measured with atoms in the cavity. This coefficient is found by an average to fits with different atom numbers in the cavity and is given by $c_2 = 0.023\ \mathrm{ms}^{-1}$.

For a Pound-Drever-Hall measurement of peak-to-peak signal amplitude V_A, the signal slope on resonance is $\mathrm{d}V/\mathrm{d}\omega = 2V_A/\kappa$, and the signal fluctuation due to shot noise is $\delta V = V_A/2\sqrt{N_s}$, where $N_s = qP_s\tau/(h\nu)$ is the expected number of sideband photons collected [2]. Here, κ is the cavity linewidth, P_s is the probe sideband power, q is the quantum efficiency of the detection path, τ is the measurement time, and ν is the laser frequency. These voltage fluctuations are converted to frequency fluctuations by dividing by the slope, so that $\delta\omega = \kappa/4\sqrt{N_s}$. Converting to atom number resolution by the factor $(\Delta N)^2 = (g_0^2/\Delta)^2(\delta\omega)^2$, and taking into account known excess noise from our detector, we obtain $(\Delta N)^2 = 0.06$ atoms variance at an averaging time of 1 ms. Independent measurements of the signal fluctuations in an empty cavity with and without stabilization of the probe laser frequency confirm that laser shot noise does not account for all of the term c_1. The term c_1 thus includes additional technical noise due to laser frequency fluctuations, which could in principle be removed with a tighter lock.

The variance due to shot noise in the atom loss process can be understood in the following way. We make a measurement of atom number N by averaging for two time periods τ to get two estimates N_1 and N_2 of atom number N, then take the measurement variance $(\mathrm{Var}(N_1 - N_2))/2$ derived from many repeated such measurements. The measurement of N_1 can be written as

$$N_1 = \frac{1}{\tau}\int_0^\tau N(t)\,\mathrm{d}t, \tag{5}$$

where

$$N(t) = N(0) - \int_0^t L(t')\,\mathrm{d}t'. \tag{6}$$

Here $L(t')$ is the instantaneous loss rate at time t'; define its expectation value as L_0. The loss in an amount of time $\mathrm{d}t'$ is a Poisson random variable with expectation value and variance both equal to $L_0\,\mathrm{d}t'$. In the limit that the total number of atoms lost during the measurement time 2τ is small, the loss at any t' is random and does not depend on t'. The expectation value and variance of the loss in a time window $\mathrm{d}t'$ are both $L_0\mathrm{d}t'$. We now write

$$\begin{aligned} N_1 - N_2 &= \frac{1}{\tau}\left[\int_0^\tau \left(N(0) - \int_0^t L(t')\,\mathrm{d}t'\right)\mathrm{d}t - \right. \\ &\left. \int_\tau^{2\tau}\left(N(0) - \int_0^t L(t')\,\mathrm{d}t'\right)\mathrm{d}t\right], \end{aligned} \tag{7}$$

which simplifies to

$$\begin{aligned} N_1 - N_2 &= \frac{1}{\tau}\left[\int_\tau^{2\tau}\int_0^t L(t')\,\mathrm{d}t'\,\mathrm{d}t \right. \\ &\quad \left. - \int_0^\tau\int_0^t L(t')\,\mathrm{d}t'\,\mathrm{d}t\right] \\ &= \frac{1}{\tau}\left[\int_\tau^{2\tau}\int_\tau^t L(t')\,\mathrm{d}t'\,\mathrm{d}t \right. \\ &\quad \left. + \int_0^\tau\int_t^\tau L(t')\,\mathrm{d}t'\,\mathrm{d}t\right]. \end{aligned} \tag{8}$$

Because the loss is random and uncorrelated, the variances of the two final terms are equal. We now make use of the relation

$$\begin{aligned} &\frac{1}{\tau}\int_0^\tau\int_t^\tau L(t')\,\mathrm{d}t'\,\mathrm{d}t \\ &= \frac{1}{\tau}\int_0^\tau\int_0^{t'} L(t')\,\mathrm{d}t\,\mathrm{d}t' \\ &= \frac{1}{\tau}\int_0^\tau t' L(t')\,\mathrm{d}t'. \end{aligned} \tag{9}$$

The variance of the final expression is easily calculated. Since the loss $L(t')$ is random and uncorrelated, the total variance is merely an integral of variances $L_0\,\mathrm{d}t'$ weighted by the factor t'^2/τ^2. Finally we obtain

$$\begin{aligned} \frac{1}{2}\mathrm{Var}(N_1 - N_2) &= \int_0^\tau \frac{t'^2}{\tau^2} L_0\,\mathrm{d}t' \\ &= \frac{1}{3}L_0\tau. \end{aligned} \tag{10}$$

Finally, this term is multiplied by the factor $(3/4)\alpha$ (see Section II of the Supplementary Information) to account for the nonuniform coupling of the atoms. Since the dominant loss processes in our system are single-atom effects, the average loss rate can be written $L_0 = N\ell_0$. Thus, the contribution to measurement variance from atom shot noise is $(1/4)\alpha\ell_0 N\tau$. For our total loss time constant of 15 ms, we calculate the coefficient of the contribution to measurement variance from atom loss to be $c_2 = (1/4)\alpha\ell_0 = 0.016\ \mathrm{ms}^{-1}$, in reasonable agreement with the fit value of $0.023\ \mathrm{ms}^{-1}$. The actual fitted value for c_2 is somewhat higher than the calculated value, most likely due to collective motional excitations within the atom cloud, which are not included in our model. Evidence for this is the even-higher variance at longer averaging times for higher atom numbers, as shown in Figure 2 in the text. The observed variances for a particular atom number therefore differ from the model predictions for our data by up to 10%. Nevertheless a simple two-parameter model demonstrates reasonable agreement with the observed measurement variance as a function of time.

[1] M. H. Schleier-Smith, I. D. Leroux, and V. Vuletić, Phys. Rev. Lett. **104**, 073604 (2010), including Supplementary Information.

[2] E. D. Black, American Journal of Physics **69**, 79 (2001).