

## Spin Squeezing on an Atomic-Clock Transition

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We generate input states with reduced quantum uncertainty (spin-squeezed states) for a hyperfine atomic clock by collectively coupling an ensemble of laser-cooled and trapped  $^{87}\text{Rb}$  atoms to an optical resonator. A quantum non-demolition measurement of the population difference between the two clock states with far-detuned light produces an entangled state whose projection noise is reduced by as much as 9.4(8) dB below the standard quantum limit (SQL) for uncorrelated atoms. When the observed decoherence is taken into account, we attain 4.2(8) dB of spin squeezing, confirming entanglement, and 3.2(8) dB of improvement in clock precision over the SQL. The method holds promise for improving the performance of optical-frequency clocks.

*Keywords:* Spin squeezing; Quantum Noise; Atomic Clock

### 1. Introduction: Projection Noise and the Standard Quantum Limit

In an atomic clock<sup>1-3</sup> or an atom interferometer,<sup>4-6</sup> the energy difference between two states is measured as a quantum mechanical phase accumulated in a given time, and the result read out as a population difference between the two states. An elegant and insightful description of the signal and noise<sup>7,8</sup> uses the angular-momentum formalism, where each individual atom  $i$  is formally associated with a spin  $s_i = \frac{1}{2}$  system, while the ensemble is described by the total spin vector  $\mathbf{S} = \sum_i \mathbf{s}_i$ . Symmetric states of the ensemble of  $N_0$  particles are then characterized by an ensemble spin quantum number  $S$  given by  $S = \frac{1}{2}N_0$ , while non-symmetric states correspond to a smaller quantum number,  $S < \frac{1}{2}N_0$ . An arbitrary symmetric state of  $N_0$  uncorrelated particles (coherent spin state, or CSS) is described by an ensemble spin vector with maximal projection  $S_1 = S$  along some direction

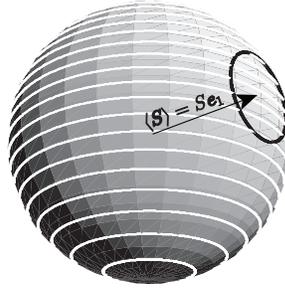


Fig. 1. Illustration of a coherent spin state (CSS). For  $N_0$  atoms, the state is represented by a circle of radius  $\sqrt{S}$  on a Bloch sphere of radius  $\sqrt{S(S+1)}$ , where  $S = N_0/2$ .

$\mathbf{e}_1$  (see Fig. 1). Note that the length of the spin vector,  $\sqrt{\langle S^2 \rangle} = \sqrt{S(S+1)}$  (in units of  $\hbar$ ) is larger than  $S$ , due to the fact that quantum mechanics imposes non-vanishing expectation values  $\langle S_2^2 \rangle, \langle S_3^2 \rangle$  for the transverse spin components  $S_2, S_3$ . Graphically, the CSS thus corresponds to the circular intersection of a sphere of radius  $\sqrt{S(S+1)}$  with the plane perpendicular to  $\mathbf{e}_1$  at distance  $S$  from the origin. The finite radius  $\sqrt{S}$  of the circle represents the angular momentum uncertainties  $\Delta S_2 = \Delta S_3 = \sqrt{S/2}$ . The possible measurement outcomes along any direction correspond to planes slicing the sphere at positions  $M = -S, -S+1, \dots, S$  relative to the origin. For a CSS in the  $xy$  equatorial plane, which is the final state of a Ramsey clock sequence, the binomial distribution of possible  $M = S_z$  values associated with the statistically independent measurement outcomes for the individual particles constitutes a fundamental source of noise that limits the precision of the measurement<sup>7-9</sup> at the standard quantum limit (SQL).

The SQL is the fundamental limit for measurements with ensembles of uncorrelated particles. However, quantum mechanics allows one to redistribute the quantum noise between different degrees of freedom by entangling the atoms in the ensemble. In Fig. 2c we represent the state of the system by a quasiprobability distribution of the noncommuting angular momentum components. The projection noise can be suppressed by reducing the quantum uncertainty in the variable of interest  $S_z$  at the expense of another variable, e.g.  $S_y$ , that is not directly affecting the experiment precision;<sup>7,8</sup> this corresponds to squeezing the circular uncertainty region of the CSS into an elliptical one. The redistribution of quantum noise for a system with a finite number of discrete states is referred to as “spin squeezing”.<sup>10</sup> A state with reduced quantum uncertainty  $S_z$  is called “number squeezed”. A state along  $x$  with reduced  $S_y$  is called “phase squeezed” (Fig. 2c iii,iv).

The two states can be converted into each other by a common rotation of all individual spins.

Note that to demonstrate spin squeezing, it is necessary not only to measure the spin noise along some direction, but also to determine the length of the spin vector  $S$ , since processes that differently affect the individual spins  $\mathbf{s}_i$  reduce the ensemble spin vector  $\mathbf{S} = \sum \mathbf{s}_i$ . The ensemble spin can be measured by determining the visibility of Rabi or Ramsey oscillations.<sup>7,8</sup> For an ensemble spin vector  $\mathbf{S}$  oriented along the  $x$  axis, a state is number squeezed or phase squeezed<sup>10-13</sup> if  $(\Delta S_z)^2 < |\langle S_x \rangle|/2$  or  $(\Delta S_y)^2 < |\langle S_x \rangle|/2$ , respectively.

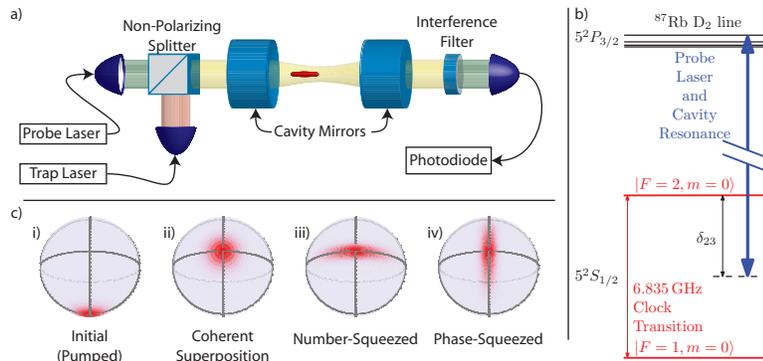
Spin squeezing requires a Hamiltonian that is at least quadratic in the spin components, or equivalently, some form of interaction between the particles. While it is possible to use interatomic collisions in a Bose-Einstein condensate (BEC) for that purpose,<sup>14,15</sup> these density-dependent interactions are difficult to control in the setting of a precision measurement. An alternative proposal is to use the collective interaction of an atomic ensemble with a mode of an electromagnetic field.<sup>16</sup> In this approach, the ensemble interacts with a far-detuned light field, resulting in an entanglement between the ensemble spin  $S_z$  and the phase or amplitude of the light field. A subsequent near-quantum-limited measurement of the light results in a conditionally spin-squeezed state of the ensemble. The word “conditionally” signifies here that the particular spin-squeezed state that is created depends on the outcome of the measurement on the light field. If one were to ignore (trace over) the state of the light, no entanglement would be evident in the atomic state.

Nevertheless, even conditionally spin-squeezed input states can improve the sensitivity of an atomic clock,<sup>17</sup> since one can use the outcome of the measurement of the light field to determine the clock phase with improved precision compared to the SQL. A perhaps even more attractive possibility is to use the information gained *during* the measurement of the light field to steer the atomic quantum state to a desired location,<sup>12,13,18</sup> thus converting the conditional into unconditional spin squeezing.

In atomic Bose-Einstein condensates, interaction-induced spin-noise reduction below the projection noise limit has been inferred from an increased noise in another spin component,<sup>19</sup> and from a lengthening in coherence time in a system with atom-number-dependent mean-field energy.<sup>15</sup> In room-temperature vapor, spin squeezing<sup>20</sup> has been achieved by absorption of squeezed light,<sup>21</sup> and two-mode squeezing has been attained by a quantum non-demolition (QND) measurement on a light beam that has

interacted with two ensembles.<sup>22</sup> A QND measurement<sup>16</sup> has been used to reduce the noise of a rotating spin in a room-temperature vapor below the projection noise limit,  $(\Delta S_z)^2 < S_0/2$ , but the length of the spin vector  $|\langle S_x \rangle|$  was not measured.<sup>23</sup> The papers by Geremia *et al.* reporting spin squeezing for atoms with  $s > \frac{1}{2}$  using a similar QND approach for cold atoms were recently retracted.<sup>24</sup> Light-induced squeezing within individual atoms of large spin  $s = 3$ , without squeezing the ensemble spin, has recently been demonstrated.<sup>25</sup>

## 2. Spin Squeezing by Optical Quantum Non-Demolition Measurement



**Fig. 2. Measurement-induced pseudo-spin squeezing on an atomic clock transition.** (a) **Setup.** A laser-cooled ensemble of  $^{87}\text{Rb}$  atoms is loaded into a far-detuned optical dipole trap inside an optical resonator. A population difference  $N$  between hyperfine clock states  $|1\rangle, |2\rangle$  produces a resonator frequency shift that is measured with a probe laser. (b) **Atomic level structure.** The resonator is tuned such that atoms in the two clock states produce equal and opposite resonator frequency shifts via the state-dependent atomic index of refraction. (c) **Preparing a squeezed input state for an atomic clock.** A number-squeezed state (iii) can be generated from a CSS along  $x$  (ii) by measurement of  $N$ . It can then be rotated by a microwave pulse into a phase-squeezed state (iv), allowing a more precise determination of the phase acquired in the free-evolution time of the atomic clock.

To prepare a spin-squeezed input state to an atomic clock, we adapt the proposal by Kuzmich, Bigelow, and Mandel<sup>16</sup> for a QND measurement of  $S_z$  with far off-resonant light.<sup>22,23</sup> By using the interaction of an optically thick ensemble with a single electromagnetic mode, the number of atoms in each of the clock states can be established beyond the projection noise limit without substantially reducing the system's coherence. For an optical depth

exceeding unity, an accurate measurement of the atomic index of refraction, which can be viewed as a homodyne measurement of the forward-scattered field, with the directly transmitted field acting as the local oscillator, can be performed faster than the scattering of photons into free space reveals the states of the individual atoms and destroys the coherence. The attainable squeezing, in terms of variances, improves as the square root of the optical depth, which is why we use an optical resonator whose finesse  $\mathcal{F} = 5600$  increases the optical depth by a factor of  $\mathcal{F}/\pi \approx 1800$ .

An ensemble of up to  $5 \times 10^4$  laser-cooled  $^{87}\text{Rb}$  atoms is trapped in a far-detuned optical dipole trap inside the optical resonator (Fig. 2). One resonator mode is tuned such that the state-dependent atomic index of refraction produces a mode frequency shift  $\omega$  that is proportional to the population difference  $N = N_2 - N_1 = 2S_z$  between the hyperfine clock states  $|1\rangle = |5^2S_{1/2}, F = 1, m_F = 0\rangle$  and  $|2\rangle = |5^2S_{1/2}, F = 2, m_F = 0\rangle$ . The frequency shift is determined from our accurately measured resonator parameters as  $d\omega/dN = 48(2)$  Hz/atom. This value is confirmed experimentally by measurement of the dual effect, namely the energy shift of the atomic levels by the intracavity light, that results in a phase shift between the clock levels of  $\phi_{12} = 250(20)$   $\mu\text{rad}$  per probe photon sent through the resonator. Given  $d\omega/dN$ , the average spin  $\langle S_z \rangle$  and variance  $(\delta S_z)^2$  are calculated from typically 50 repeated transmission measurements of a probe pulse tuned to the slope of the resonator mode. Light pulses of duration  $T = 50$   $\mu\text{s}$ , much longer than the resonator decay time of  $\tau = \kappa^{-1} = 158$  ns, containing  $10^5$  to  $10^6$  photons traverse the atom-resonator system and are detected with an overall quantum efficiency of  $Q_e = 0.43(4)$ . A frequency stabilization system for probe laser and resonator ensures that the probe transmission noise is close to the photocurrent shot-noise limit. One of the experimental challenges is to stabilize the resonator length sufficiently well to resolve the mode shift due to atomic projection noise, typically a few kHz out of a 1 MHz resonator linewidth, while using light levels that lead only to a modest decoherence between the clock states.

We verify experimentally the projection noise level for the coherent spin state (CSS) of an uncorrelated ensemble<sup>7,8,23</sup> by measuring probe transmission for  $p = 5 \times 10^5$  photons transmitted on average through the resonator. To reduce the effect of trap loading fluctuations, we perform a CSS preparation and measurement sequence (consisting of optical pumping into state  $|1\rangle$ ,  $\pi/2$  pulse, and measurement of  $S_z$ ) twice with the same loaded atoms and determine the variance  $(\delta S_z)^2$  between the two measurements. As a function of (effective) atom number  $N_0$ , projection noise is characterized

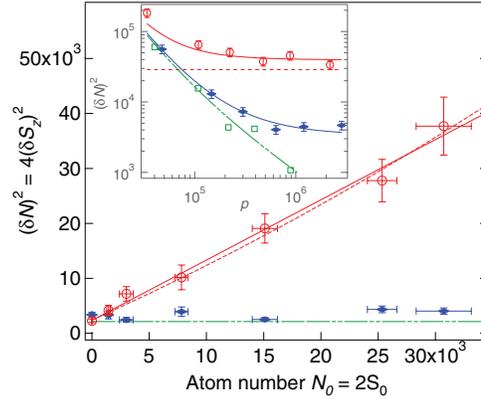


Fig. 3. **Projection noise limit and spin noise reduction.** The measured spin noise for an uncorrelated state (CSS, open circles) agrees with the theoretical prediction  $(\Delta S_z)^2 = S_0/2$ , with negligible technical noise (solid and dashed lines). Our measurement of  $S_z$  at photon number  $p = 5 \times 10^5$  has an uncertainty  $(\delta S_z)^2$  (solid diamonds) substantially below the SQL. **Inset:** Dependence of spin measurement variance  $(\delta S_z)^2 = (\delta N)^2/4$  on probe photon number  $p$  for  $N_0 = 3 \times 10^4$ . With increasing photon number, the measurement uncertainty (solid diamonds) drops below the projection noise level  $(\Delta S_z)_{\text{CSS}}^2 = aS_0/2$  (dashed line), while the variance measured for independently prepared CSSs (open circles) approaches  $(\Delta S_z)_{\text{CSS}}^2$ . Also shown is the technical noise without atoms, expressed as an equivalent spin noise (open squares).

by a variance  $(\delta S_z)^2 \propto N_0$ , while for technical noise  $(\delta S_z)^2 \propto N_0^2$ . (In a standing-wave resonator with spatially-modulated atom-cavity coupling, we define the effective atom number  $N_0 = \frac{4}{3}N_{\text{tot}}$  as the ideal projection noise variance for  $N_{\text{tot}}$  atoms evenly distributed along the cavity axis.) Unlike other experiments,<sup>20,22,23</sup> we have a reliable and accurate absolute calibration of the atom number via the resonator shift and can not only test the linear dependence  $(\delta S_z)^2 = aN_0$  but also compare the slope  $a$  to a calculated value that takes into account the spatially inhomogeneous coupling between the trapped atoms and the probe light. Fig. 3 shows the dependence of variance  $(\delta N)^2 = 4(\delta S_z)^2$  on atom number  $N_0 = 2S_0$  (open circles). The fitted slope  $a_f = 1.1(1)$  is slightly higher than the calculated value  $a_c = 0.93(1)$  due to technical noise at large atom number. If we fix  $a = a_c = 0.93$  and fit this quadratic technical noise, we find a small contribution  $(\delta S_z)_{\text{tech}}^2 = 6(4) \times 10^{-6} N_0^2 \ll N_0$  (dashed curve in Fig. 3). This confirms that we have a system dominated by projection noise, and quantitatively establishes the SQL.

We prepare a state with conditionally reduced noise  $(\Delta S_z)^2$  simply by measuring  $S_z$  for a CSS along  $x$  with a photon number sufficiently large to

resolve  $S_z$  beyond the CSS variance  $(\Delta S_z)_{\text{CSS}}^2 = S_0/2$ . This measurement with variance  $(\delta S_z)^2$  prepares a state with a random but known value of  $S_z$  whose quantum uncertainty is  $(\Delta S_z)^2 = (\Delta S_z)_{\text{CSS}}^2 (\delta S_z)^2 / ((\Delta S_z)_{\text{CSS}}^2 + (\delta S_z)^2)$ . (Throughout this report,  $\delta S_z$  refers to a measured standard deviation, while  $\Delta S_z$  denotes a quantum uncertainty for the pure or mixed state that we are preparing.  $\Delta S_z$  differs from  $\delta S_z$  because it includes the prior knowledge that the state is initially prepared as a coherent state along  $x$ . The distinction has little effect for strong squeezing, but for weak squeezing ensures that the initial quantum uncertainty is taken into account correctly.<sup>26</sup>) The faithfulness of the state preparation is verified with a second measurement, and we plot the variance of the two measurements  $(\delta N)^2 = 4(\delta S_z)^2$  vs. atom number  $N_0$  in Fig. 3 (solid diamonds). While at low atom number the measurement noise exceeds the SQL due to photon shot noise and some technical noise (dash-dotted line in Fig. 3), at higher atom number  $N_0 = 3 \times 10^4$  we achieve a 9.4(8) dB suppression of spin noise below the SQL.

The inset to Fig. 3 shows  $(\delta N)^2$  vs. average transmitted photon number  $p$  at fixed  $N_0 = 3 \times 10^4$  for the CSS as well as for the reduced-uncertainty state. At low  $p$ , photon shot noise prevents observation of the spin projection noise level (dashed line). For large  $p$  the observed noise for the CSS (open circles) reaches a plateau that corresponds to spin projection noise, while the squeezing measurement localizes the value of  $S_z$  to better than the projection noise (solid diamonds). For photon numbers  $p \leq 5 \times 10^5$  the squeezing measurement is close to the technical noise without atoms (open squares).

Having established that we can prepare states with spin noise  $\Delta S_z$  below the projection limit, we need to verify whether the system remains sufficiently coherent to guarantee entanglement. The prepared state is spin squeezed, and thereby entangled,<sup>10</sup> if  $\zeta_{\text{KU}} = 2(\Delta S_z)^2 / (a|\langle \tilde{S} \rangle|) < 1$ , where  $\tilde{S}$  is the ensemble spin in the  $xy$ -plane.<sup>10</sup> Fig. 4 shows, as a function of photon number in the preparation pulse, the normalized spin-noise  $(\Delta S_z)^2 / (\Delta S_z)_{\text{CSS}}^2$  (open diamonds), and the measured clock contrast  $\mathcal{C} = |\langle \tilde{S} \rangle| / S_0$  (open squares). Shown also is the squeezing parameter  $\zeta_{\text{KU}}$  obtained by dividing the observed spin-noise reduction by  $\mathcal{C}$ , demonstrating that we have achieved 4.2(8) dB of spin squeezing for  $p = 3 \times 10^5$ . We emphasize that in this analysis we use the full observed noise, including photon shot noise and all technical noise, and all contrast reduction, including contrast loss due to the resonator locking light (evident as finite contrast  $\mathcal{C}_{\text{in}} = 0.7$  for no probe pulse ( $p = 0$ ) in Fig. 4). We find that  $\mathcal{C}_{\text{in}}$  can

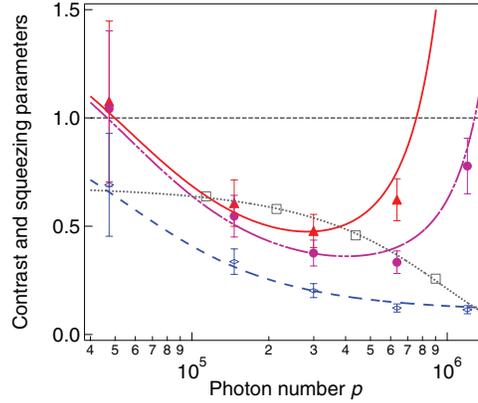


Fig. 4. **Spin noise reduction, loss of contrast, and spin squeezing.** The reduction of normalized spin noise  $(\Delta S_z)^2 / (\Delta S_z)_{\text{CSS}}^2$  (open diamonds and dashed curve) below unity is accompanied by a loss of coherence observable as a reduced contrast  $\mathcal{C}$  (open squares and dotted curve) in a Ramsey clock sequence. From these two measurements, we can deduce two squeezing parameters (see text),  $\zeta_{KU} = 2(\Delta S_z)^2 / (a|\langle \tilde{S} \rangle|)$  (solid circles and dash-dotted curve), which characterizes the entanglement of the squeezed state, and  $\zeta_W = 2(\Delta S_z)^2 S_{\text{in}} / (a|\langle \tilde{S} \rangle|^2)$  (solid triangles and solid curve), which characterizes the squeezing-induced improvement in clock performance.

be improved compared to Fig. 4 by choosing a larger detuning from atomic resonance for the lock light. (In Fig. 4 that detuning is  $\sim 14$ GHz.) The contrast reduction due to the probe light is probably due to a motion-induced fluctuation of the differential light shift between the clock states, and can be reduced by cooling the atoms further. The fundamental lower limit for contrast loss, set by the scattering of photons into free space, should allow the squeezing parameter  $\zeta_{KU}$  to approach the 9 dB spin noise reduction observed at our highest probe photon numbers  $p > 1 \times 10^6$ . If technical noise can be reduced further, the fundamental limit associated with scattering is set by the optical depth  $OD$  of the sample<sup>27</sup> and for our present parameters ( $OD = 5 \times 10^3$ ) amounts to  $\sim 18$  dB of spin squeezing.

The usefulness of the state for precision measurements is quantified by the more stringent parameter<sup>7,8</sup>  $\zeta_W = 2(\Delta S_z)^2 S_{\text{in}} / (a|\langle \tilde{S} \rangle|^2) < 1$ . This expression is easily understood as a reduction of the squared noise-to-signal ratio  $(\Delta S_z)^2 / |\langle \tilde{S} \rangle|^2$  relative to its value in the unsqueezed coherent state  $a/(2S_{\text{in}})$ . For our system, the ensemble without squeezing has  $S_{\text{in}} = S_0 \mathcal{C}_{\text{in}}$ , yielding  $\zeta_W = \zeta_{KU} \mathcal{C}_{\text{in}} / \mathcal{C}$ . This parameter, also plotted in Fig. 4, shows an improvement in clock precision of 3.2(8) dB.

### 3. Outlook

We have verified that the prepared number-squeezed state can be converted into a phase-squeezed state by a  $\pi/2$  microwave pulse about  $\langle \mathbf{S} \rangle$ , and used as an input state to a Ramsey type atomic clock. Note that the spin vector precesses through many revolutions in a typical atomic clock. Therefore in an optical-transition atomic-ensemble clock,<sup>2,3</sup> fractional frequency accuracies of  $10^{-16}$  can be achieved with fairly modest absolute phase accuracies<sup>2</sup> of  $\Delta\phi \sim 10^{-2}$ , which can readily be improved by the squeezing technique investigated here. It should also be possible to apply this squeezing technique to atom interferometers<sup>6</sup> and other precision experiments with atomic ensembles. We believe that most of the technical limitations in the current experiment, such as remaining technical transmission noise due to imperfect laser-resonator frequency stabilization, and contrast loss due to spatially inhomogeneous light shifts, can be overcome in the near future, allowing for squeezing near the fundamental limit set by the sample's optical depth. Since even the latter can be improved by simply loading more atoms into the trap, we believe that 15 to 20 dB of spin squeezing should not represent an unrealistic goal for the near future.

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The group of E. Polzik has independently and simultaneously achieved results<sup>28</sup> similar to ours<sup>29</sup> in a Mach-Zehnder interferometer. In this meeting, M. Oberthaler and coworkers report spin squeezing in a Bose-Einstein condensate by atomic interactions in a multiple-well potential.<sup>30</sup>

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